LINKÖPING UNIVERSITY

Advanced Computational Statistics – Spring 2025
Assignment for Lecture 6

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Perform the solutions individually and send your report **until June 7** by email to me. Try to keep this deadline. However, if you have problems with the June-7-deadline, there will be a final deadline on September 30 for all assignments. Please send me one **pdf-file** with your report, and additionally, please send me your code in one separate **plain-text file**. **The following two problems are mandatory.**

Problem 6.1

There are several significance tests which can be used to test if a sample is normally distributed:

$$H_0: \text{ sample is } N(\mu, \sigma^2)\text{-distributed for some } \mu \text{ and } \sigma.$$

Your task here is to investigate the Shapiro–Wilk test used with a significance level $\alpha = 0.05$ for a sample size of $n = 200$.

a. Simulate the type I error of the test by generating standard normally distributed samples and applying the Shapiro-Wilk test. Use an appropriate number s of repetitions and give a reason why you have chosen it. Determine the size of the Monte Carlo error for your estimated type I error.

b. Simulate the power of the test if data has a distribution with density $f(x)$ for the different densities below. For this, generate random variables following the target density $f(x)$ with an appropriate method (you can use the available standard functions in your programming language for generation of random variables). Use a number s of repetitions which gives reasonable precision of the result and argue for that.

   - $f(x) = \frac{8}{\pi}\sqrt{x(1-x)}$ if $0 \le x \le 1$ and $f(x) = 0$ for other x (Beta(1.5,1.5)-distribution),
   - $f(x) = \frac{3}{8}x^2$ if $0 \le x \le 2$ and $f(x) = 0$ for other x,
   - $f(x) = const \cdot \exp(-x^2)(2 + \cos(7\pi x)), x \in I\!R$.

Problem 6.2

We consider the initial iteration of adaptive rejection sampling (without squeezing). The one-dimensional density $f(x)$ which we want to generate is here the standard normal distribution, i.e. $f(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$.

a. We start with $T_2 = \{x_1, x_2\}$, where $x_2 = -x_1 = t > 0$. Compute an envelope $e^*(x)$ (defined via the tangents in x_i) and $e(x) = \exp(e^*(x))$. Plot $f(x)$ together with $e(x)$ for one or two t.

b. Compute the waste generated by this envelope. Optimize t such that the waste is as small as possible.

c. Use now $T_4 = \{-t_2, -t_1, t_1, t_2\}$ with $0 < t_1 < t_2$ as initial grid. Compute the envelope and the waste. Optimize (t_1, t_2) such that the waste is as small as possible. Plot $f(x)$ together with $e(x)$ for the optimal choice of t_1, t_2.